On the vibrational convective instability of a horizontal, binary-mixture layer with Soret effect

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A theoretical examination is made of the mechanical quasi-equilibrium stability of a horizontal, binary-mixture layer with Soret effect in the presence of a high-frequency vibrational field. The boundaries of the layer are assumed to be rigid, isothermal and impermeable. The axis of vibration is longitudinal. The study is based on the system of equations describing the behaviour of mean fields. The conditions of quasi-equilibrium are formulated. A linear stability analysis for normal modes is carried out. In the limit of long-wave disturbances the regular perturbation method is used with the wavenumber as a small parameter. For the case of an arbitrary wavenumber, the calculations are made using straight forward numerical integration. The boundaries of stability and the critical disturbance characteristics are determined for representative parameter values. Different instability mechanisms and forms are discussed.

1. Introduction

It is known that the vibration of a cavity filled with fluid having temperature inhomogeneity can generate some regular mean flows even in the absence of a static gravity field, i.e. under conditions of pure weightlessness – the phenomenon of thermovibrational convection (Gershuni & Zhukhovitsky 1979, 1981). In the limit of high frequency and small amplitude the method of averaging can be effectively applied to investigate thermovibrational convection. In the theory of convection this method was first used by Zen'kovskaya & Simonenko (1966) to study the effect of high-frequency vertical vibration on the stability of a horizontal fluid layer heated from below.

Under certain conditions mechanical quasi-equilibrium is possible, i.e. the state with zero mean velocity and generally non-zero oscillatory component. Some examples of quasi-equilibrium configurations with results of linear convective stability analysis were presented by Gershuni & Zhukhovitsky (1979, 1981) and later by Braverman (1984, 1987a, b).

In the case of pure weightlessness only the specific thermovibrational mechanism is responsible for instability excitation. In the presence of static gravity both

thermogravitational and thermovibrational mechanisms occur. The configuration corresponding to a horizontal layer heated either from below or from above under longitudinal vibration is considered by Braverman *et al.* (1984). The experimental results given by Zavarykin, Zorin & Putin (1988) are in good agreement with the theoretical ones. The works of Gershuni, Zhukhovitsky & Kolesnikov (1986, 1990), Gershuni & Zhukhovitsky (1988) and Gershuni *et al.* (1989, 1992) are devoted to different aspects of linear and nonlinear instability caused by static gravity and vibration in the presence of internal heat generation, and the experimental results reported by Kozlov & Shatunov (1988) agree fairly well with the theoretical prediction.

In our short review we consider only the papers which tackle the problem of quasi-equilibrium vibrational stability. Additional references concerning this problem can be found in Chernatynsky, Gershuni & Monti (1993).

All the papers cited above dealt with a one-component fluid. For a binary mixture with inhomogeneous concentration, additional buoyancy forces due to gravity and vibrational fields appear as well as an additional dissipative mechanism caused by diffusion (and thermodiffusion). The problem is thus more complicated. To the best of our knowledge, only two papers have appeared on the problem of vibrational stability of binary mixtures. Zen'kovskaya (1981) studied a horizontal layer of the mixture with physically non-realistic boundary conditions. Braverman (1987a, b) considered the case of weightlessness when the three vectors – temperature gradient, concentration gradient and the axis of vibration – belong to common plane, perpendicular to the layer; the solution corresponding to a long wave mode is obtained. The Soret effect is not taken into account either by Zen'kovskaya (1981) or by Braverman (1987). But it is not just this reason which makes us to study the stability of the mixture with Soret effect.

The stability of the solidification front of alloys is modified by constitutional supercooling. The Soret effect must be taken into account as it can modify the concentration gradient in the liquid near the liquid–solid interface as stated by Van Vaerenbergh & Legros (1990). It has been shown that, depending on the sign of the thermodiffusion coefficient, morphological stability may be increased or decreased, oscillatory regimes of convection are also foreseen (see e.g. Hurle 1983 and Van Vaerenbergh *et al.* 1995).

It is expected when processing materials under reduced gravity conditions that the remixing of components by buoyancy-induced convection will be strongly reduced. But the gravity level on board orbital laboratories is not constant: the orientation changes of the vehicles induce low-frequency gravity variation and the activities on board (crew motions, motors, etc.) create g-jitter fluctuating randomly in magnitude and direction.

Furthermore, during some experiments on board Spacelab they have deal with plane layers of binary mixtures in the presence of high-frequency vibrations. It appeared interesting to study the influence of thermodiffusion (the Soret effect) on the threshold of convection in the presence of vibration and a residual static gravity field. The next question is how to control the onset of convection in binary mixtures by means of vibrations.

In the present work we study the double-diffusive situation, i.e. the convective instability of a horizontal binary mixture plane layer with Soret effect subject to a static gravitational field and longitudinal high-frequency vibration. It is demonstrated that under these circumstances the mechanical quasi-equilibrium is possible. The analysis of its linear stability with respect to two-dimensional disturbances of normal mode type is performed. In the limiting case free of vibration, the problem reduces to



FIGURE 1. Geometry and coordinate system.

the known one (Legros, Platten & Poty 1972; Platten & Legros 1984, containing an extensive bibliography). In the opposite limit of zero gravity, only one mechanism of excitation (vibrational) is operative and the problem reduces to vibrational convective stability of a binary mixture with Soret effect in weightlessness. We concentrate also on situations when instability is caused by both mechanisms – vibrational and gravitational.

In §2 we describe the problem and write down the basic system of equations governing free convection of a binary mixture with Soret effect under vibration in the framework of the standard Boussinesq approximation. In §3 the system of equations for mean fields is obtained by applying the averaging technique. Non-dimensional parameters of the problem are introduced. In §4 the conditions of quasi-equilibrium are formulated in general form and in reference to the case considered. Section 5 is devoted to the statement of the stability problem. The spectral problem for amplitudes of two-dimensional-normal disturbances is formulated. In §6 we consider the limit of the long-wave mode and develop a regular perturbation method with the wavenumber as a small parameter for expansion. The numerical results for the case of arbitrary values of the wavenumber are presented and discussed in §7.

2. Description of the problem; basic equations

Let us consider an infinite plane horizontal layer of a binary mixture with Soret effect. The layer is confined between two rigid isothermal and impermeable planes z = 0 and z = h (the geometry and coordinate system are shown in figure 1). The temperature of the lower plane is constant and equal to Θ , the temperature of the upper plane is also constant and it is taken as the reference point. Thus $\Theta > 0$ corresponds to the case of heating from below while $\Theta < 0$ refers to heating from above. Both boundaries are impermeable, and there is no externally imposed difference of concentration. The only reason for concentration inhomogeneity is the Soret effect (de Groot 1945; de Groot & P. Mazur 1984; Tyrrell 1961). The fluid layer and its boundaries undergo linear harmonic oscillations in the x-axis direction. The object is to study the possibility of mechanical quasi-equilibrium and to carry out the linear analysis of its stability.

Assuming the validity of Boussinesq approximation one must neglect the mechanical compressibility. Then the deviations of temperature T and concentration C (expressed as a mass fraction) from their standard constant values \overline{T} and \overline{C} are relatively small and the equation of state has the form

$$\rho = \bar{\rho}(1 - \beta_1 T - \beta_2 C), \qquad (2.1)$$

where ρ is the fluid density and $\bar{\rho}$ is its standard constant value; $\beta_1 > 0$ is the thermal expansion coefficient and β_2 is the concentrational coefficient of the density. Taking *C* as the concentration of the lighter component, $\beta_2 > 0$.

We introduce the appropriate (non-inertial) coordinates connected with the oscillating system (cavity with fluid). To write the equation of motion in the appropriate system of coordinates it is necessary to replace the gravity acceleration g by

$$\boldsymbol{g} \to \boldsymbol{g} + b\Omega^2 \cos \Omega t \boldsymbol{n}. \tag{2.2}$$

Here b is the displacement amplitude, Ω is the angular frequency and **n** is the unit vector along the axis of vibration. The second term in the right-hand side of (2.2) is purely vibrational acceleration. Then we have the equation of motion in the form

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p + v\nabla^2 \boldsymbol{v} + g(\beta_1 T + \beta_2 C)\boldsymbol{\gamma} - (\beta_1 T + \beta_2 C)b\Omega^2 \cos\Omega t\boldsymbol{n}.$$
(2.3)

Here v is the velocity, v is the kinematic viscosity of the fluid, γ is the unit vector directed vertically upward.

Now let us write down the diffusion equation. Taking into account the thermodiffusional Soret effect, the flux of the lighter component is given by

$$\mathbf{j} = -\bar{\rho}D(\nabla C + \alpha \nabla T), \qquad (2.4)$$

where D is the coefficient of diffusion and α is the thermodiffusional ratio. It is evident that in the case of the normal Soret effect, the lighter component moves towards higher temperature and $\alpha < 0$.

Note that for many liquid solutions, $\beta_1 T$ is of the same order of magnitude as $\beta_2 C$ induced by the Soret effect.

At the level of the linear approximation for the concentration distribution induced at the steady state by the Soret effect, (2.1) can also be written as

$$\nabla \rho(T,C) = \left[\frac{\partial \rho}{\partial T} + \frac{\partial \rho}{\partial C}\alpha\right] \nabla T.$$

For $\alpha < 0$ ('anomalous effect'), the Soret effect is thus competing against the effect of the thermal expansion and situations exist with $\beta_1 T = -\beta_2 C$, i.e. that a non-isothermal system may have a constant density.

On the other hand, for $\alpha > 0$ ('normal effect') the Soret effect contributes to reinforce the density gradient induced by the thermal expansion.

In concentrated solutions, this contribution of the Soret effect to the density profile is generally not small with respect to the thermal expansion effect and has dramatic consequences on the hydrodynamic stability of the system (see e.g. Legros *et al.* 1972 and Hurle & Jakeman 1971)

Hereafter we shall assume that characteristic differences of temperature and concentration are not large, and the coefficients D and α are independent of temperature and concentration. Then the diffusion equation is of the form

$$\frac{\partial C}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} C = D(\boldsymbol{\nabla}^2 C + \alpha \boldsymbol{\nabla}^2 T).$$
(2.5)

The heat transport and continuity equations are written as

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} T = \kappa \nabla^2 T, \qquad (2.6)$$

$$\nabla \cdot \boldsymbol{v} = \boldsymbol{0}, \tag{2.7}$$

where κ is the heat diffusivity coefficient.

The equations (2.3), (2.5)-(2.7) with appropriate initial and boundary conditions

describe thermal and concentrational free convection in the appropriate system of coordinates under static gravity and vibration.

3. The system of equations for mean fields

In the limiting case of high frequency and small amplitude of vibration the method of averaging can be applied effectively to study the phenomena of vibrational convection. This technique is widely used in different areas of physics and mechanics (Landau & Lifshitz 1988). According to this method we subdivide each field into two parts: the first part varies slowly with time (the characteristic time is large with respect to the vibration period) and the second one varies quickly with time (the characteristic time is of the order of magnitude of the vibration period). Thus we have the decompositions

$$\boldsymbol{v} = \tilde{\boldsymbol{v}} + \boldsymbol{v}', \quad p = \tilde{p} + p', \quad T = \tilde{T} + T', \quad C = \tilde{C} + C'.$$
 (3.1)

Here \tilde{v} , \tilde{p} , \tilde{T} and \tilde{C} are the 'slow' (averaged) fields, and v', p', T' and C' are the 'quick' (small) parts. Now substitute (3.1) into the equation of motion (2.3), separate the 'quick' parts and simplify the equation for the oscillatory part as much as possible. Retaining only the main terms we obtain

$$\frac{\partial \boldsymbol{v}'}{\partial t} = -\frac{1}{\rho} \nabla p' - b\Omega^2 \cos \Omega t \boldsymbol{n} (\beta_1 \tilde{T} + \beta_2 \tilde{C}).$$
(3.2)

In the left-hand side of (3.2) we keep only the term with the derivative with respect to (quick) time, while in the right-hand side we neglect the viscous term. It means that the frequency of vibration must be high (but below acoustic), so the period of vibration τ must be small with respect to all characteristic hydrodynamic times:

$$\tau \ll \min(h^2/\nu, h^2/\kappa, h^2/D). \tag{3.3}$$

Then in the left-hand side of (3.2) we neglect the nonlinear term $(v' \cdot \nabla)v'$ with respect to $\partial v' / \partial t$. It means that the displacement amplitude must be small in the sense

$$b \ll \frac{h}{\beta_1 \Theta}.\tag{3.4}$$

Here $\beta_1 \Theta$ is a non-dimensional parameter which is small in the framework of the Boussinesq approach. Thus the amplitude of displacement may be even larger than the characteristic scale.

Finally, in the right-hand side of (3.2) we neglect the gravitational buoyancy forces for the 'quick' component of the flow. Thus we have

$$\frac{g}{b\Omega^2}\frac{b}{h}\beta_1 \Theta \ll 1. \tag{3.5}$$

The criteria (3.3)–(3.5) will be discussed below.

Now we decompose the vector $\mathbf{n}(\beta_1 \tilde{T} + \beta_2 \tilde{C})$ in (3.2) as follows:

$$\boldsymbol{n}(\beta_1 \tilde{T} + \beta_2 \tilde{C}) = \boldsymbol{w} + \boldsymbol{\nabla}\boldsymbol{\varphi},\tag{3.6}$$

where w is its solenoidal part and $\nabla \varphi$ is its potential one. Substituting (3.6) into (3.2) and separating solenoidal parts we have

$$\frac{\partial \boldsymbol{v}'}{\partial t} = -b\Omega^2 \cos \Omega t \boldsymbol{w}. \tag{3.7}$$

Integrating over the quick time yields

$$\mathbf{v}' = -b\Omega\sin\Omega t\mathbf{w}.\tag{3.8}$$

We see that the additional slow variable w is not only the solenoidal part of the vector $\mathbf{n}(\beta_1 \tilde{T} + \beta_2 \tilde{C})$; it is also the slow variation with time of the amplitude of the oscillatory velocity component (in some other scale).

An analogous procedure must be applied to equations (2.5) and (2.6). The result is

$$T' = -b\cos\Omega t(\boldsymbol{w}\cdot\nabla\tilde{T}), \quad C' = -b\cos\Omega t(\boldsymbol{w}\cdot\nabla\tilde{C}).$$
(3.9)

One could see that the criteria (3.3)–(3.5) are obtained by means of estimations of oscillating parts of the fields v', T' and C' according to (3.8), (3.9):

$$v' \sim b\Omega \beta_1 \Theta, \qquad T' \sim C' \sim b\beta_1 \Theta^2/h.$$

Thus the relations among oscillatory and averaged parts of the fields are determined. The formulae (3.8) and (3.9) give the solution of the 'closing' problem. The last step is to substitute the decomposition (3.1) with v', T' and C' determined by (3.8), (3.9) into the basic system (2.3), (2.5)–(2.7) and perform the averaging procedure. After integration over the quick time we obtain the closed system of equations for mean fields. With tildes omitted we have

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p + v\nabla^2 \boldsymbol{v} + g(\beta_1 T + \beta_2 C)\boldsymbol{\gamma} + \frac{1}{2}b^2\Omega^2(\boldsymbol{w} \cdot \nabla)\left[(\beta_1 T + \beta_2 C)\boldsymbol{n} - \boldsymbol{w}\right],$$
(3.10)

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} T = \kappa \nabla^2 T, \qquad (3.11)$$

$$\frac{\partial C}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} C = D(\boldsymbol{\nabla}^2 C + \alpha \boldsymbol{\nabla}^2 T), \qquad (3.12)$$

$$\nabla \cdot \boldsymbol{v} = \boldsymbol{0}, \tag{3.13}$$

$$\nabla \cdot \boldsymbol{w} = 0, \qquad \nabla \times \boldsymbol{w} = \nabla(\beta_1 T + \beta_2 C) \times \boldsymbol{n}.$$
 (3.14)

Notice that in the limit of high frequency and small amplitude the effect of vibration is determined by the product $b\Omega$.

Boundary conditions for velocity, temperature and concentration must be imposed in accordance with the physical statement of the problem. When establishing the boundary condition for the additional 'slow' variable w we should keep in mind that the viscous term in equation (3.2) for the oscillatory velocity component v' is neglected (in this approach the dynamic Stokes layer is not resolved). Hence we cannot formulate the non-slip condition for the oscillatory part of the velocity, and the non-overflow condition will be appropriate, i.e. $w_n|_F = 0$ where F is the rigid boundary.

In the case of a plane horizontal layer bounded by rigid isothermal and impermeable planes we have the following set of boundary conditions:

at
$$z = 0$$
 and $z = h$: $v = 0$, $w_z = 0$, $\frac{\partial C}{\partial z} + \alpha \frac{\partial T}{\partial z} = 0$,
at $z = 0$: $T = \Theta$,
at $z = h$: $T = 0$.
(3.15)

The governing system (3.10)–(3.14) with appropriate boundary conditions determines the mean flow in the presence of high-frequency vibration. Using (3.8), (3.9) we may also find the oscillatory parts of the fields. We introduce non-dimensional variables with the help of the following scales: h for distance, h^2/κ for time, κ/h for velocity, Θ for temperature, $\beta_1 \Theta/\beta_2$ for concentration, $\beta_1 \Theta$ for w and $\rho \kappa^2/h^2$ for pressure. Thus we obtain the system of governing equations for non-dimensional variables

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla p + Pr\nabla^2 \boldsymbol{v} + RaPr(T+C)\boldsymbol{\gamma} + Ra_v Pr(\boldsymbol{w} \cdot \nabla) \left[(T+C)\boldsymbol{n} - \boldsymbol{w} \right], \quad (3.16)$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} T = \boldsymbol{\nabla}^2 T, \qquad (3.17)$$

$$\frac{\partial C}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} C = L \boldsymbol{e} \boldsymbol{\nabla}^2 (C - \varepsilon T), \qquad (3.18)$$

$$\nabla \cdot \boldsymbol{v} = \boldsymbol{0},\tag{3.19}$$

$$\nabla \cdot \boldsymbol{w} = 0, \qquad \nabla \times \boldsymbol{w} = \nabla (T+C) \times \boldsymbol{n}.$$
 (3.20)

The non-dimensional form of boundary conditions is

at
$$z = 0$$
 and $z = 1$: $v = 0$, $w_z = 0$, $\frac{\partial C}{\partial z} - \varepsilon \frac{\partial T}{\partial z} = 0$,
at $z = 0$: $T = 1$,
at $z = 1$: $T = 0$.
(3.21)

The problem formulated involves the following non-dimensional parameters: the Rayleigh number Ra, the vibrational analogue of the Rayleigh number Ra_v , the Prandtl and Lewis numbers Pr and Le, the parameter of the Soret effect ε . These parameters are defined by

$$Ra = \frac{g\beta_1\Theta h^3}{v\kappa}, \quad Ra_v = \frac{(b\Omega\Theta h\beta_1)^2}{2v\kappa}, \quad \varepsilon = -\frac{\alpha\beta_2}{\beta_1}, \quad Pr = \frac{v}{\kappa}, \quad Le = \frac{D}{\kappa}.$$
 (3.22)

The Rayleigh number Ra is positive when the system is heated from below and negative when heated from above. The vibrational Rayleigh number Ra_v is always positive in accordance with its definition. The Soret parameter ε is positive in the case of the normal effect and negative in the case of the anomalous effect.

4. Mechanical quasi-equilibrium

Under certain conditions mechanical quasi-equilibrium is possible, i.e. the state at which the mean velocity is zero but the oscillatory part, in general, exists. To determine the necessary conditions of mechanical quasi-equilibrium we refer to (3.16)– (3.20). Equating velocity to zero we seek the steady-state distributions of temperature, concentration, pressure and w. Applying the curl procedure to both sides of (3.16) we obtain the equations for quasi-equilibrium fields T_0, C_0, w_0 :

$$\left\{ \begin{array}{l} \nabla(T_0 + C_0) \times [Ra\gamma - Ra_v \nabla(w_0 \cdot \boldsymbol{n})] = 0, \\ \nabla^2 T_0 = 0, \quad \nabla^2 C_0 = 0, \\ \nabla \cdot w_0 = 0, \quad \nabla \times w_0 = \nabla(T_0 + C_0) \times \boldsymbol{n} \end{array} \right\}$$
(4.1)

with appropriate boundary conditions.

The quasi-equilibrium state exists only in some special cases of geometry, conditions of heating and vibration. It can be seen that for a horizontal plane layer with boundary conditions (3.21) the quasi-equilibrium state occurs and has the following structure:

$$T_0 = T_0(z),$$
 $C_0 = C_0(z),$ $w_{0x} = w_0(z),$ $w_{0y} = w_{0z} = 0,$ (4.2)

where the quasi-equilibrium profiles have the form

$$T_0 = 1 - z, \quad \frac{dC_0}{dz} = -\varepsilon, \quad w_0 = -(1 + \varepsilon) \left(z - \frac{1}{2}\right).$$
 (4.3)

In this case the oscillatory flow in quasi-equilibrium is longitudinal. We suppose that there is no net oscillatory flow:

$$\int_{-1}^{1} w_0 \mathrm{d}z = 0. \tag{4.4}$$

It is easy to see that distributions (4.2), (4.3) satisfy all the conditions (4.1) and (3.21). Thus the mechanical quasi-equilibrium state in the situation considered exists at arbitrary values of Ra and Ra_v .

5. Formulation of the stability problem

To study the linear stability of mechanical quasi-equilibrium we consider perturbed fields:

$$v, T_0 + T', C_0 + C', p_0 + p', w_0 + w'.$$
 (5.1)

After linearization we obtain the following system of equations for small disturbances:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p' + Pr\nabla^2 \boldsymbol{v} + RaPr(T' + C')\boldsymbol{\gamma} + Ra_v Pr\left\{ (\boldsymbol{w}_0 \cdot \nabla) \left[(T' + C')\boldsymbol{n} - \boldsymbol{w}' \right] + (\boldsymbol{w}' \cdot \nabla) \left[(T_0 + C_0)\boldsymbol{n} - \boldsymbol{w}_0 \right] \right\},$$
(5.2*a*)

$$\frac{\partial T'}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} T_0 = \boldsymbol{\nabla}^2 T', \qquad (5.2b)$$

$$\frac{\partial C'}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} C_0 = Le \boldsymbol{\nabla}^2 (C' - \varepsilon T'), \qquad (5.2c)$$

$$\nabla \cdot \boldsymbol{v} = \boldsymbol{0}, \tag{5.2d}$$

$$\nabla \cdot w' = 0, \qquad \nabla \times w' = \nabla (T' + C') \times n$$
 (5.2e)

with homogeneous boundary conditions

at
$$z = 0$$
 and $z = 1$: $\mathbf{v} = 0$, $T' = 0$, $w'_z = 0$,
$$\frac{\partial C'}{\partial z} - \varepsilon \frac{\partial T'}{\partial z} = 0.$$
 (5.3)

By analogy with the problem of thermovibrational convective stability in a layer filled with a one-component fluid it might be expected that two-dimensional disturbances are the most unstable ones. So we consider two-dimensional-disturbances $v(v_x, 0, v_z)$, $w'(w'_x, 0, w'_z)$, T', C' and p' which are independent of the coordinate y. We eliminate the disturbance of pressure p' and introduce the stream functions Ψ and F for solenoidal vectors v and w' respectively:

$$v_x = \frac{\partial \Psi}{\partial z}, \qquad v_z = -\frac{\partial \Psi}{\partial x}; \qquad w'_x = \frac{\partial F}{\partial z}, \qquad w'_z = -\frac{\partial F}{\partial x}.$$
 (5.4)

Now introduce disturbances of normal mode type:

$$(\Psi, T', C', F) = (\varphi(z), \theta(z), \xi(z), f(z)) \exp(-\lambda t + ikx).$$
(5.5)

Here k is the wavenumber, λ is the decay rate, $\varphi(z), \theta(z), \xi(z)$ and f(z) are the amplitudes.

Substituting (5.5) into the system of equations for disturbances gives the system of equations for amplitudes

$$-\lambda \mathscr{D}\varphi = \Pr \mathscr{D}^{2}\varphi + ikRaPr(\theta + \xi) + ikRa_{v}Pr(1 + \varepsilon)(\theta + \xi - f'), -\lambda\theta - ik\varphi = \mathscr{D}\theta, -\lambda\xi - ik\varepsilon\varphi = Le\mathscr{D}(\xi - \varepsilon\theta), \mathscr{D}f = \theta' + \xi'.$$
(5.6)

Here the prime indicates differentiation with respect to the transversal coordinate z and \mathscr{D} is the operator $\mathscr{D} = d^2/dz^2 - k^2$.

Using (5.3) one can obtain the boundary conditions for the amplitudes:

at
$$z = 0$$
 and $z = 1$: $\varphi = \varphi' = 0$, $\theta = 0$, $f = 0$,
 $\xi' - \varepsilon \theta' = 0$. (5.7)

The system of equations (5.6) with boundary conditions (5.7) corresponds to the spectral amplitude problem with the decay rate λ as an eigenvalue and with amplitudes as eigenvector components. The characteristic values of the decay rate depend on all the parameters of the problem:

$$\lambda = \lambda(Ra, Ra_v, Pr, Le, \varepsilon, k).$$
(5.8)

The decay rate λ is complex in general, $\lambda = \lambda_r + i\lambda_i$ because the spectral amplitude problem is not self-conjugated. If $\lambda_i = 0$ the stability boundary is determined by the condition $\lambda = 0$ (the monotonic mode of instability). If $\lambda_i \neq 0$ the stability boundary is determined by the condition $\lambda_r = 0$ and in this case λ_i is the frequency of neutral oscillation (the oscillatory mode of instability). Note that in accordance with our averaging approach the frequency λ_i must be small with respect to the frequency of vibration.

6. The limiting case of the long-wave mode

We may expect that because of the impermeability condition for concentration the long wave instability with k = 0 plays a substantial role for some range of parameters (physically this means that the disturbance wavelength is much larger than the thickness of the layer). Then the decay rate spectrum and stability boundary for long-wave disturbances may be determined by applying the regular perturbation method with the wave number k as a small parameter. The solution is constructed in the form of power expansions:

$$\varphi = \varphi_0 + k\varphi_1 + k^2\varphi_2 + \cdots,
 \theta = \theta_0 + k\theta_1 + k^2\theta_2 + \cdots,
 \xi = \xi_0 + k\xi_1 + k^2\xi_2 + \cdots,
 f = f_0 + kf_1 + k^2f_2 + \cdots,
 \lambda = \lambda_0 + k\lambda_1 + k^2\lambda_2 + \cdots.$$
(6.1)

Substituting these expansions in the system of amplitude equations (5.6) and equating the terms with the same order of k we obtain the systems of successive approximations (the boundary conditions for each order coincide with the set (5.7)).

For the zero order we have

$$\left. \begin{array}{l} -\lambda_{0}\varphi_{0} = Pr\varphi_{0}^{\mathrm{iv}}, \\ -\lambda_{0}\theta_{0} = \theta_{0}'', \\ -\lambda_{0}\xi_{0} = Le(\xi_{0}'' - \varepsilon\theta_{0}''), \\ f_{0} = \theta_{0}' + \xi_{0}'. \end{array} \right\}$$
(6.2)

It can be seen that all the levels of the decay rate spectrum correspond to damping disturbances except for one level which is of 'concentrational type' and neutral:

$$\lambda_0 = 0, \qquad \varphi_0 = 0, \qquad \theta_0 = 0, \qquad f_0 = 0, \qquad \xi_0 = \text{ const },$$
 (6.3)

Here const can be chosen equal to unity, for example, under appropriate normalization.

For the first order we obtain the non-homogeneous system of equations:

$$\begin{array}{c}
\varphi_{1}^{\text{iv}} = -\mathrm{i}\xi_{0} \left[Ra + (1+\varepsilon)Ra_{v}\right], \\
\theta_{1}^{"} = 0, \\
Le(\xi_{1}^{"} - \varepsilon\theta_{1}^{"}) = -\xi_{0}\lambda_{1}, \\
f_{1}^{"} = \theta_{1}^{'} + \xi_{1}^{'}.
\end{array}$$
(6.4)

The solvability condition for this system can be derived by integrating both sides of the third equation with respect to z from 0 to 1, taking into account the impermeability boundary condition. This yields $\lambda_1 = 0$. Thus we obtain the first-order solution:

$$\lambda_1 = 0, \quad \theta_1 = 0, \quad \xi'_1 = 0, \quad f_1 = 0, \varphi_1 = -\frac{i\xi_0}{24} \left[Ra + (1+\varepsilon)Ra_v \right] z^2 (1-z)^2.$$
(6.5)

For the second order we have

$$\begin{array}{c}
\varphi_{2}^{\mathrm{iv}} = -\mathrm{i}\xi_{1} \left[Ra + (1+\varepsilon)Ra_{v} \right], \\
\theta_{2}^{\prime\prime} = -\mathrm{i}\varphi_{1}, \\
Le(\xi_{2}^{\prime\prime} - \varepsilon\theta_{2}^{\prime\prime}) = \xi_{0}(Le - \lambda_{2}) - \mathrm{i}\varepsilon\varphi_{1}, \\
f_{2}^{\prime\prime} - \theta_{2}^{\prime} - \xi_{2}^{\prime} = 0.
\end{array}\right\}$$
(6.6)

The condition of solvability for this non-homogeneous system may be obtained by integrating the third equation with respect to z from 0 to 1 :

$$\xi_0(Le - \lambda_2) - i\varepsilon \int_0^1 \varphi_1 dz = 0.$$
(6.7)

After substituting φ_1 from (6.5) we have

$$\lambda_2 = Le - \frac{\varepsilon}{720} \left[Ra + (1+\varepsilon)Ra_v \right].$$
(6.8)

It is readily seen that the decay rate λ is real, so the long-wave instability is of monotonic type. The stability boundary is determined from the condition $\lambda_2 = 0$.



FIGURE 2. Boundaries of long-wave instability. Regions of instability are hatched. (a) $Ra_v = 0$, (b) Ra = 0.

This leads to

$$Ra + (1+\varepsilon)Ra_v = \frac{720Le}{\varepsilon}.$$
(6.9)

When $Ra_v = 0$, i.e. vibration is absent, then

$$Ra = \frac{720Le}{\varepsilon}.$$
(6.10)

Thus the long-wave instability exists in the case of normal Soret effect when heated from below and in the case of anomalous Soret effect when heated from above (see figure 2a). Note that the dimensional density gradient in equilibrium state is equal to

$$\frac{\mathrm{d}\rho}{\mathrm{d}z} = \frac{\tilde{\rho}\beta_1\Theta}{h}(1+\varepsilon).$$

From this follows that the anomalous Soret effect with $\varepsilon = -1$ corresponds to the equilibrium state without vertical density stratification. The instability in this case is caused by the double-diffusive mechanism.

In the opposite limiting case when Ra = 0 (pure weightlessness) we obtain from (6.9)

$$Ra_v = \frac{720Le}{\varepsilon(1+\varepsilon)}.$$
(6.11)

The instability in this case is caused by the vibrational mechanism and exists in the range $\varepsilon > 0$ (normal Soret effect) and $\varepsilon < -1$ (strong anomalous Soret effect). Within the interval $-1 < \varepsilon < 0$ the long-wave instability is absent (see figure 2*b*); formally in this interval the critical vibrational Rayleigh number is negative.

In the general case $(Ra \neq 0, Ra_v \neq 0)$ the relation (6.9) shows that the boundary of stability in the plane (Ra, Ra_v) is linear.

To find out whether the long-wave mode with k = 0 is the most unstable or not it is necessary to compare the above analytical results with numerical ones obtained for the case of an arbitrary value of the wavenumber k.

7. Numerical results and discussion

To solve the complete spectral amplitude problem formulated by (5.6) and (5.7) for arbitrary (finite) values of the wavenumber the numerical technique was applied.



FIGURE 3. The stability diagrams in the plane Ra_m/ε for the case $Ra_v = 0$. The solid lines represent the monotonic cellular modes, the dashed lines correspond to the oscillatory cellular modes and the dotted lines to the monotonic long wave modes: curve 1, Le = 1; 2, Le = 1.5; 3, Le = 0.5; 4, Le = 0.01.

The straightforward numerical integration of the system of equations for amplitudes by means of the Runge-Kutta-Merson method was used in combination with the shooting procedure. This enables one to find the eigenvalue λ from the spectral problem numerically and, in particular, to determine the boundaries of stability and the characteristics of critical disturbances, such as the wavenumbers of the most unstable modes and the critical frequencies of oscillatory modes.

We have explored a large set of parameters. Some computational results of the instability characteristics for representative values of the parameters Ra_v , Ra and Le are presented. We use the following definition of the Lewis number: $Le = D/\kappa$, It has been varied from 0.01 up to 1.5.

Let us recall here that for gaseous mixtures generally $Le \sim 1$, as D is of the same order of magnitude as κ . On the other hand, for liquid mixtures, usually $Le \ll 1$, for instance for solutions like water-salt, water-sugar, water-ethanol, $Le \sim 0.01$.

The results are presented in figures 3–9.

Since for gaseous mixtures both Le > 1 and Le < 1 are possible because $D \sim \kappa$, we will refer to 'the case of a gaseous mixture' if $Le \sim 1$. On the other, since for liquid mixtures usually $Le \leq 1$ (the thermal diffusivity is normally much larger than the molecular diffusivity) we will speak in such conditions of the 'case of a typical liquid mixture'.

7.1. The case of no vibration $(Ra_v = 0)$

First consider the results for the simplest case when vibration is absent, $Ra_v = 0$, and only the gravitational mechanism is responsible for instability excitation. In



FIGURE 4. The stability diagrams for the case Ra = 0 (weightlessness). The nomenclature and numeration of lines are the same as in figure. 3.

figure 3 the minimal critical values of the Rayleigh number Ra_m are presented as functions of the non-dimensional Soret parameter ε for a few combinations of the other parameters (the minimization is made with respect to the wavenumber k). The solid lines correspond to the boundaries of monotonic instability of cellular form (the minima of the neutral curves are at $k_m \neq 0$); the dashed lines correspond to the oscillatory cellular critical modes, and the dotted lines to the long-wave monotonic modes. The results on long-wave instability presented in figures 3–9 by dotted lines are also obtained numerically. They coincide perfectly with the analytical ones (6.10).

Lines 1 correspond to the Lewis number value Le = 1 (for example the case of a gaseous mixture with Pr = Sc = 1; here Sc = v/D is the Schmidt number). It is obvious physically that under such conditions no oscillatory instability exists since the characteristic diffusion time and the heat diffusivity time coincide. Lines 2 correspond to Le = 1.5 (Le > 1), for instance for a model gaseous mixture with Pr = 0.75 and Sc = 0.5. In this case an oscillatory instability is possible but the monotonic one is more unstable. Lines 3 correspond to Le = 0.5 (Le < 1), e.g. corresponding to a model gaseous mixture with Pr = 0.75 and Sc = 1.5.

It is seen that there is a competition between monotonic and oscillatory instability modes in the range of the anomalous Soret effect ($\varepsilon < 0$). Lines 4 correspond to a typical liquid binary mixture (like salt-water solution), namely Pr = 6.7 and Sc = 677($Le \approx 0.01$, $Le \ll 1$). In this case monotonic and oscillatory modes are also competing at $\varepsilon < 0$; in practically the entire range $\varepsilon < 0$ the oscillatory mode is the most unstable when the layer is heated from below.

Note that all the stability lines in the region $Ra_m > 0$ intersect the axis Ra_m at the point $Ra_m = 1708$ which corresponds to the Rayleigh–Bénard instability boundary



FIGURE 5. Critical values of Rayleigh number Ra_m versus non-dimensional Soret parameter ε for fixed Le = 1 and several values of vibrational Rayleigh number: 1, $Ra_v = 0$; 2, $Ra_v = 200$; 3, $Ra_v = 1000$; 4, $Ra_v = 2129$; 5, $Ra_v = 3000$.

for a one-component fluid. We recall that in our statement of the problem it has been supposed that there is no imposed concentration difference, and the only concentration difference which exists is caused by the Soret effect. Thus the transition to the case of one-component liquid corresponds to $\varepsilon \rightarrow 0$.

Furthermore by virtue of the system of amplitude equations (5.6) the stability boundaries for monotonic modes do not depend individually on the Prandtl and Schmidt numbers but only on their combination $Pr/Sc \equiv Le$.

The picture presented in figure 3 displays the following features:

(i) destabilization at $\varepsilon > 0$ which is highly pronounced in the case of a liquid mixture. This destabilization of the quasi-equilibrium is physically evident as in this case of normal Soret effect ($\varepsilon > 0$) the lightest component of the mixture migrates in the direction of the hot plate, i.e. down; (see e.g. Legros *et al.* 1972). Thus the non-stable stratification of the liquid layer increases and the critical temperature difference necessary for the instability excitation, decreases;

(ii) stabilization and in some cases the existence of oscillatory modes at $\varepsilon < 0$;

(iii) the existence of monotonic instability modes at $\varepsilon < 0$ and Ra < 0, i.e. when the layer is heated from above.

The data describing the wavenumber k_m of the most unstable mode and the neutral frequency λ_{im} are presented at the top of the figure.

7.2. The case of weightlessness (Ra = 0)

Let us discuss further the opposite limiting case, Ra = 0, which corresponds to a state of pure weightlessness. Here only the vibrational mechanism of instability excitation



FIGURE 6. Critical values of Rayleigh number Ra_m versus ε for a gaseous mixture with Le = 1.5 (Le > 1) and different values of vibrational Rayleigh number: 1, $Ra_v = 0$; 2, $Ra_v = 2129$; 3, $Ra_v = 5000$.

is operative. In figure 4 the critical values of the vibrational Rayleigh number Ra_{vm} (minimized with respect to the wavenumber k) and the critical disturbance characteristics are plotted versus the non-dimensional Soret parameter ε . The nomenclature and numeration of lines are the same as in figure 3.

When $\varepsilon = 0$ the problem reduces to that of a one-component fluid. In this case the critical value of the vibrational Rayleigh number is $Ra_{vm} = 2129$ and the critical wavenumber is $k_m = 3.23$ (Gershuni & Zhukhovitsky 1979). It is seen that for $\varepsilon > 0$ destabilization takes place. On the other hand in the range $-1 < \varepsilon < 0$ the quasi-equilibrium is stabilized because of the Soret effect. The influence of the Soret mechanism on stability is very strong, especially in the case of a liquid mixture (note the use of a logarithmic scale along the Ra_v -axis). For $\varepsilon < -1$ instability also exists and the long-wave mode plays an important role. As mentioned above (see figure 2b), there is no long-wave instability inside the gap $-1 < \varepsilon < 0$. It can be seen also that in this region, cellular instability of both monotonic and oscillatory character can coexist.

7.3. Combined case

Let us consider a few examples of calculations corresponding to the general case when both parameters Ra and Ra_v are not equal to zero and both physical mechanisms of instability excitation coexist. Figure 5 demonstrates the effect of vibration on stability in the case of Le = 1 when only monotonic instability is present. The region of mechanical quasi-equilibrium stability is situated on the plane (ε , Ra_m) between the pairs of lines with the same numbers: 1 - 1, 2 - 2, etc. The line 4 corresponds to the



FIGURE 7. Critical values of Ra_m versus ε for a gaseous mixture with Le = 0.5 (Le < 1) and different values of vibrational Rayleigh number: 1, $Ra_v = 0$; 2, $Ra_v = 2129$; 3, $Ra_v = 5000$. The oscillatory mode is the most unstable in the region $\varepsilon < 0$.

value $Ra_v = 2129$ and intersects the Ra_{vm} -axis just at the coordinates origin; recall that this value is related to the instability threshold when static gravity is absent. The instability in the range $\varepsilon < 0$ is mainly due to the long-wave modes (dotted lines). It is interesting to examine line 2 ($Ra_v = 200$): in the region $\varepsilon < 0$ the only finite interval of this stability boundary is connected with the cellular mode. The main feature of the family of curves presented in figure 5 is that the stability region in the plane (ε , Ra_m) reduces as the parameter Ra_v increases. This effect of destabilization is definitely related to the increasing role of the vibrational mechanism of excitation.

The situation presented in figure 6 is qualitatively close to that just described for figure 5. Here Le = 1.5 (Le > 1) and the monotonic mode is the most unstable for all parameters considered.

The results presented in figure 7 correspond to the gaseous mixture with Pr = 0.75and Sc = 1.5. The Lewis number in this case is Le = 0.5 (Le < 1) and so the oscillatory instability is possible in the region of the anomalous Soret effect $\varepsilon < 0$. It is interesting to note that the oscillatory form of instability may exist when the system is heated from above and Ra_v is large (line 3, $Ra_v = 5000$).

Figure 8 refers to a typical case of a liquid mixture (Pr = 6.7, Sc = 677). It is seen that the instability is of monotonic cellular character only in the range of small positive or negative ε . The long-wave instability plays the main role when either $\varepsilon > 0$ and takes not too small values or $\varepsilon < 0$ when heating from above. In addition the oscillatory form of instability is developed at $\varepsilon < 0$ when the layer is heated from below (stability regions are confined between the curves with the same numbers).

Finally we present the stability diagram for a specific binary liquid mixture: water



FIGURE 8. Critical values of Ra_v versus ε for a liquid mixture with $Le \approx 0.01$ and different values of vibrational Rayleigh number: 1, $Ra_v = 0$; 2, $Ra_v = 2129$; 3, $Ra_v = 5000$.



FIGURE 9. The stability diagram on the plane (Ra, Ra_v) for liquid mixture (water-ethanol). I, region of stability; II, region of oscillatory cellular instability; III, region of monotonic long-wave instability.

(90%w)-ethanol (10%w). In accordance with the data presented in the literature and some estimations, the parameters can be fixed as follows: Pr = 11.3; Sc = 1100; $\varepsilon = -4.5 \times 10^{-2}$. For these values of the parameters the stability diagram (or regimes) is shown in figure 9 in the plane (Ra, Ra_v) as a result of minimization with respect to the wavenumber. The line $\varepsilon = 0$, corresponding to the case of a one-component liquid, is given for comparison. In region I the mechanical quasi-equilibrium is stable. The dashed line is the boundary of oscillatory instability; thus region II is the region of oscillatory convection. It should be emphasized that a relatively small negative Soret effect (the denser component is migrating toward the hot side) has little influence on the instability threshold as compared to the case of a pure fluid. However it changes the instability character qualitatively from monotonic to oscillatory (this can be seen also in figures 3 and 4 where the transition from a monotonic mode to the oscillatory one takes place at very small negative values of ε : see lines 4). The dotted line corresponds to the long-wave monotonic instability, so in the region III steady long wave convection exists. Figure 9 also depicts schematically the forms of neutral curves in the plane (k, Ra) for each region.

8. Conclusions

In this paper we have investigated theoretically the linear stability of a plane horizontal layer of a binary mixture with Soret effect subject to static gravity and longitudinal high-frequency vibration. The study is based on the closed system of equations for mean fields. The conditions of quasi-equilibrium are determined. The spectral amplitude problem for small normal disturbances is formulated. The regular method of perturbation with the wavenumber k as a small parameter is developed to study the behaviour of long-wave modes and to determine the stability boundary at k = 0. The eigenvalue problem for arbitrary k is solved numerically. The limiting cases of the absence of vibration and of weightlessness are considered. Furthermore situations involving both mechanisms of instability excitation, gravitational and vibrational, are analysed numerically for representative values of parameters. Three types of instability are distinguished: monotonic cellular, monotonic long wave and oscillatory cellular. The analysis allows one to establish the following characteristic features. In the case of the normal Soret effect only monotonic instability takes place and thermodiffusion plays a destabilizing role; in the case of the anomalous Soret effect all the instability modes compete. The threshold of instability due to the vibrational mechanism strongly depends on ε .

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